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The Pile Simulator:

A Network Analogue to the 2-Group Analysis.*

I The Network and Its Equations

Consider the situation shown in Fig. 1 in which a current density J feeds current to a conducting surface (conductance per square of m_1) at a point whose potential is V_1. The current flows away (1) in the plane with surface current density $\vec{K_1}$, (2) through an anisotropic conductor ("specific" conductance n_1 per unit area) with a current density I_1 to a second surface and (3) direct to ground through an anisotropic conductor ("specific" conductance kn_1 per unit area) with a current density I_3. The current I_1 is dissipated (1) in the second surface with current density $\vec{K_2}$ and (2) to ground through an anisotropic conductor ("specific" conductance n_2 per unit area) with a current density I_2.

The equations governing the electrical relations here are:

$$m_1 \nabla V_1 = -\vec{K_1}, \qquad m_2 \nabla V_2 = -\vec{K_2}$$

$$J = \nabla \cdot \vec{K_1} + I_1 + I_3, \quad I_1 = \nabla \cdot \vec{K_2} + I_2$$

$$n_1 (V_1 - V_2) = I_1, \quad n_2 V_2 = I_2$$

$$n_1 k V_1 = I_3$$

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* This analysis has been made in connection with the design of a pile simulator using point-by-point amplification. The first such device was set up at Oak Ridge by E. J. Wade and J. Simpson.

which lead to

$$-\nabla^2 V_1 + \frac{n_1}{m_1}(1+k) V_1 = \frac{n_1}{m_1} V_2 + \frac{1}{m_1} J \qquad (1)$$

$$-\nabla^2 V_2 + \frac{n_1+n_2}{m_2} V_2 = \frac{n_1}{m_2} V_2 \qquad (2)$$

A network analogue to Fig. 1 is shown in Fig. 2. The meshes are square, S units on a side. Accordingly, the current fed into each upper level junction must be JS^2. The planar network, consisting of the conductances g_1, has a surface resistance g_1 per square, and similarly for the lower net. Accordingly,

$$g_1 = m_1 , \quad g_2 = m_2$$

The interlevel conductances h_1 simulate a continuous medium of conductance h_1/S^2, so that

$$n_1 = h_1/S^2 , \quad n_2 = h_2/S^2$$

With these substitutions Eqs. (1) and (2) become

$$-\nabla^2 V_1 + \frac{h_1}{S^2 g_1}(1+k) V_1 = \frac{h_1}{S^2 g_1} V_2 + \frac{1}{g_1} J \qquad (3)$$

$$-\nabla^2 V_2 + \frac{h_1+h_2}{S^2 g_2} V_2 = \frac{h_1}{S g_2} V_1 \qquad (4)$$

II The Network as an Analogue

In the pile the two-group formulation of the neutron behavior

is

$$-D_1\nabla^2\Psi_1 + A_1\Psi_1 = B_1\Psi_2 \text{ , fast}$$

$$-D_2\nabla^2\Psi_2 + A_2\Psi_2 = B_2\Psi_1 \text{ , intermediate}$$

where

$$\Psi = n v \Sigma_s , \qquad D = \frac{\overline{1}}{3\overline{\Sigma_{tr}}\,\overline{\Sigma_s}} \qquad (4.5), (5)$$

$$A_1 = \frac{\overline{\Sigma_{a1}}}{\overline{\Sigma_{s1}}} - \nu\frac{\overline{\Sigma_{f1}}}{\overline{\Sigma_{s1}}} + \frac{\xi}{\Delta} , \quad B_1 = \nu\frac{\overline{\Sigma_{f2}}}{\overline{\Sigma_{s2}}} \qquad (6) , (7)$$

$$A_2 = \frac{\overline{\Sigma_{a2}}}{\overline{\Sigma_{s2}}} , \quad B_2 = \frac{\xi}{\Delta} \qquad (8) , (9)$$

For ease in satisfying boundary conditions later we shall use

$$\Phi = \frac{\overline{1}}{\overline{\Sigma_s}}\Psi \equiv \lambda_s \Psi \quad \text{(defining } \lambda_s\text{)}$$

so that the group equations become

$$-\nabla^2\Phi_1 + \frac{A_1}{D_1}\Phi_1 = \frac{\lambda_{s1}B_1}{\lambda_{s2}D_1}\Phi_2 \qquad (5)$$

$$-\nabla^2\Phi_2 + \frac{A_2}{D_2}\Phi_2 = \frac{\lambda_{s2}}{\lambda_{s1}}\frac{B_2}{D_2}\Phi_1 \qquad (6)$$

We now establish the correspondence between network and group equations. We put*

$$V_1 = m\Phi_1 , \quad V_2 = m\mu\Phi_2 \qquad (6.5)$$

---

* m and n have new meanings here unrelated to m_1 , n_1 , etc.

leaving m and μ free for later specification. In the simulator JS^2 will be proportional to V_2, so we also write

$$J = \frac{nV_2}{S_2} = \frac{mn\mu}{S^2}\,\Phi_2$$

where n is the transfer conductance of the amplifier.

By equating coefficients it follows that

$$\frac{h_1\,(1 \neq k)}{S^2\,g_1} = \frac{A_1}{D_1}\;;\quad \frac{\mu(n \neq h_1)}{S^2\,g_1} = \frac{\lambda_1\,B_1}{\lambda_2\,D_1} \qquad (10),\ (11)$$

$$\frac{h_1 \neq h_2}{S^2\,g_2} = \frac{A_2}{D_2}\;;\quad \frac{h_1}{\mu S^2 g_2} = \frac{\lambda_2\,B_2}{\lambda_1\,D_2} \qquad (12),\ (13)$$

The quantity m does not appear. Having chosen a multiplying medium as expressed by its properties in the right members above, having set k arbitrarily for the time being, and having selected the scale factor between upper and lower mesh voltages, we can choose a conductance g_1 of convenient value and determine in turn h_1 (from 10), g_2 from (13), h_2 from (12) and n from (11). The formal analogy between network and the 2-group pile equations has thus been established.

Two discrepancies appear, however, when we proceed from the mathematical symbols to the physical facts. A good reflector has no absorption and no fission. This means that $B_1 = A_2 = 0$. The first discrepancy lies in Eq. (12) which requires that

$$h_2 = -h_1$$

which is inconvenient. If, actually, we make $h_2 = 0$, then the network is simulating a virtual medium in which, using Eq. (13)

$$\frac{\overline{\Sigma_{a2}}}{\Sigma_{s2}} = A_2 = D_2 \frac{h_1}{S^2 g_2} = \mu \frac{\lambda_2}{\lambda_1} \frac{\xi}{\Delta} \qquad (14)$$

The virtual medium has an absorption which becomes smaller if μ is chosen smaller and which, numerically, will turn out to be negligible, so that this virtual medium is a permissible substitute for the real medium.

The second discrepancy arises from Eq. (11). Combining it with Eq. (10) and substituting for A_1 and B_1 from Eqs. (6) and (7) leads to

$$n = \frac{S^2 g_1}{D_1} \left[\frac{\lambda_1}{\mu \lambda_2} \frac{\Sigma_{f2}}{\Sigma_{s2}} + \frac{\nu \Sigma_{f1}}{(1+k)\Sigma_{s1}} \right. \qquad (14.5)$$

$$\left. \frac{\Sigma_{a1}}{(1+k)\Sigma_{s1}} - \frac{\xi}{(1+k)\Delta} \right]$$

As there will be no amplifier feed-back in the reflector, we shall have $n = 0$. Let us achieve this in our virtual reflector by letting

$$\Sigma_{a1} = \Sigma_{f1} = 0, \quad \nu \frac{\Sigma_{f2}}{\Sigma_{s2}} = \mu \frac{\lambda_2}{\lambda_1} \frac{\xi}{\Delta} \qquad (15)$$

This virtual fission is again a property which is under control through μ.

Combining Eqs. (14) and (15) gives

$$\frac{\nu \dfrac{\overline{\Sigma_{f2}}}{\Sigma_{s2}}}{\dfrac{\overline{\Sigma_{a2}}}{\Sigma_{s2}}} = \frac{1}{1+k} \qquad (16)$$

Since we shall find that k is of the order of unity, we see that the virtual reflector yields high energy neutrons to about the same extent that it absorbs low energy ones. Thus, the discrepancy is partially self-compensating although tending to increase the fast flux and to decrease the slow flux slightly. In addition, as noted above, each of the virtual properties can be kept small.

With the preceding discussion in mind we can now tabulate Eqs. (10) to (13) with particular reference to core (subscript C) and reflector (subscript R). In the core we shall have no need of direct conduction from the high network to ground so that for it $k_C = 0$. Then

$$\frac{h_{1C}}{s^2 g_{1C}} = \frac{A_{1C}}{D_{1C}} ; \qquad \frac{h_{1R}\,(1 \neq k_R)}{s^2 g_{1R}} = \frac{A_{1R}}{D_{1R}} \qquad (17C, R)$$

$$\frac{n \neq h_{1C}}{g_{1C}} = \frac{\lambda_{1C}}{\mu\lambda_{2C}} \frac{B_{1C}}{D_{1C}} ; \qquad n = 0 \qquad (18C, R)$$

$$\frac{h_{1C}}{s^2 g_{2C}} = \frac{\mu\lambda_{2C}}{\lambda_{1C}} \frac{B_{2C}}{D_{2C}} ; \qquad \frac{h_{1R}}{s^2 g_{2R}} = \frac{\mu\lambda_{2R}\, B_{2R}}{\lambda_{1R}\, D_{2R}} \qquad (19C, R)$$

$$\frac{h_{2C} \neq h_{1C}}{s^2 g_{2C}} = \frac{A_{2C}}{D_{2C}} ; \qquad h_{2R} = 0 \qquad (20C, R)$$

III The Boundary Conditions Between Core and Reflector.

The boundary conditions between core and reflector which are to be fulfilled are:

$$\Phi_{(1,2)C} = \Phi_{(1,2)R} \qquad (21)$$

$$\left(D_{(1,2)C} \,/\, \lambda_{s(1,2)C}\right) \nabla \vec{\Phi}_{(1,2)C} = \left(D_{(1,2)R} \,/\, \lambda_{s(1,2)R}\right) \nabla \vec{\Phi}_{(1,2)R} \quad (22)$$

Between core net and reflector net the conditions that are automatically fulfilled are:

$$V_{(1,2)C} = V_{(1,2)R} \quad (23)$$

$$K_{(1,2)C} = K_{(1,2)R} \quad (24)$$

Equation (21) is valid by virtue of Eq. (6.5).

Two simple types of interface between networks are shown in plan in Fig. 3 at A and B.

In considering the second boundary condition we note that the electrical analogue to grad $\vec{\Phi}$ is grad V, which, in the network, becomes a finite voltage difference. In order that it apply at the boundary, this difference should be taken across the boundary and not to either side of it, but this invades the medium with which equality is to be established. The solution of this difficulty lies in the stratagem of supposing each region independently to reach across the boundary just for the purpose of determining the change in V which is centered at the boundary. This is illustrated in Fig. 4A. The actual boundary arrangement is shown in Fig. 4B.

Considering the core network, it is seen that

$$(\text{const}) \times \frac{\partial \vec{\Phi}_C}{\partial x} \simeq \frac{\Delta V}{2S} = - \frac{K' + K''}{2S \, \delta C} \quad (25)$$

By splitting the vertical conductors into halves the vertical currents

are halved also with the result that the boundary current K is

$$K = \frac{K' + K''}{2} \simeq - (\text{const}) \times S g_C \frac{\partial \Phi_C}{\partial x}$$

For the reflector also,

$$K \simeq - (\text{const}) \times S g_R \frac{\partial \Phi_R}{\partial x} \qquad (26)$$

and when the two nets are combined as at Fig. 4B we have

$$g_C \frac{\partial \Phi_C}{\partial x} \simeq g_R \frac{\partial \Phi_R}{\partial x}$$

For this to simulate Eq. (22) we must have

$$\frac{g_{(1,2)C}}{g_{(1,2)R}} = \frac{D_{(1,2)C}}{D_{(1,2)R}} \frac{\lambda_{s(1,2)R}}{\lambda_{s(1,2)C}} \qquad (27)$$

This establishes the relation between the networks representing the two media when the boundary lies as at Fig. 3A.

The boundary of Fig. 3B can be analyzed in the same way and leads to the same result.

Figs. 3A' and 3B' are also consistent with Eq. (27), the only difference in the analysis being that components of the flux gradients enter the argument.

The application of Eq. (27) to Eqs. (17C) and (17R) establishes a relation between the conductance levels of the core and reflector net-

works because it fixes h_{1C}/h_{1R}:

$$\frac{h_{1C}}{h_{1R}} = (1 + k_R)\ \frac{\lambda_{s1R}}{\lambda_{s1C}}\ \frac{A_{1C}}{A_{1R}} \tag{28}$$

But Eqs. (19C) and (19R) can be used in the same way, giving

$$\frac{h_{1C}}{h_{1R}} = \frac{\lambda_{s1R}}{\lambda_{s1C}}\ \frac{B_{2C}}{B_{2R}}$$

To make these two equations consistent we still have a face choice of K:

$$k_R = \frac{A_{1R}\ B_{2C}}{A_{1C}\ B_{2R}} - 1$$

$$= \frac{\nu \dfrac{\Sigma_{f1C}}{\Sigma_{s1C}} - \dfrac{\Sigma_{a1C}}{\Sigma_{s1C}}}{\dfrac{\xi_C}{\Delta} - \left(\nu \dfrac{\Sigma_{f1C}}{\Sigma_{s1C}} - \dfrac{\Sigma_{a1C}}{\Sigma_{s1C}}\right)} \tag{29}$$

This leaves 2 free choices, (1) the general impedance level of the networks as determined by the initial choice of g_{1C}, say, and (2) the simulation factor ratio μ. The first is determined by the requirements of amplification and sufficient time constant and the second by the need to keep the virtual absorption in the core (Eq. (14)) small.

IV Interpretation of Critical Condition in Network.

Analysis of Eqs. (17) to (20) and Eq. (29) reveals that a value of ν must be "built-into" the network. This means that the value of n

which makes the simulator critical cannot be interpreted as a ν_c. It can be interpreted in another way. B_{1C} is the only nuclear property coefficient which involves $\overline{\Sigma_{f2C}} / \Sigma_{s2C}$ and it appears only in Eq. (18C) for n. We can, therefore, look upon variation of n as effecting variation of the low velocity fission cross-section in accordance with Eq. (14.5) applied to the core:

$$n_C = \frac{s^2_{g1C}}{D_{1C}} \left(\frac{\lambda_{1C}}{\mu \lambda_{2C}} \nu \frac{\overline{\Sigma_{f2C}}}{\Sigma_{s2C}} - \frac{A_{1C}}{1 + k} \right) \qquad (30)$$

Another approach is to make compromises which avoid "building in" ν. This requires the outright assumption that $\Sigma_{f1} = 0$ which makes A_{1C} independent of ν. This would mean that the simulator would merely simulate the thermal pile case with adjusted constants. It might be that relative changes in ν_c would have fair accuracy. But in this case the simulator would certainly yield a bad velocity distribution of neutrons.

V A Network Design.

For an octagonal pile, 17" between parallel faces, with 18 unit meshes in that distance, $S = \frac{17"}{18} = 2.4$ cm., $S^2 = 5.76$.

For the nuclear properties R. Ehrlich and I have used the Table of Nuclear Constants of GE-RE-1, "A Multi-Group Method for Computing Critical Masses of Intermediate Piles," 5/9/47. Groups A, B, and C were combined to form Group 1 here and D and E gave Group 2.

The tabulation follows:

	Core					Reflector				
Group	λ_s	D	$\frac{\Sigma_a}{\Sigma_s}$	ξ	α	λ_s	D	$\frac{\Sigma_a}{\Sigma_s}$	ξ	Δ
1	3.26	3.27	.0134	.207	.115	2.17	1.67	0	207	8
2	2.16	1.33	.085	0	.180	1.44	.747	0	0	(7)

$$\Sigma_a = (1 + \alpha)\,\Sigma_f$$

The D's of Eqs. (17) to (20) appear in the table. For the other constants in those equations, we then have

$$A_{1C} = .0134\left(1 - \frac{2.5}{1.115}\right) + \frac{.207}{8} = -.0157 + .0259 = .0102$$

$$A_{2C} = .085$$

$$B_{1C} = 2.5\,\frac{.085}{1.18} = .18\,; \qquad B_{2C} = .0259$$

$$A_{1R} = .0259\,;\ A_{2R} = 0\,;\ B_{1R} = 0\,;\ B_{2R} = .0259$$

From Eq. (29) $\quad k = 1.54$

We can now work through Eqs. (17) to (20) if we assume an h_{1C} and a μ.

Let $h_{1C} = .25 \times 10^{-6}$, $h_{1C}^{-1} = 4.0\ \Omega$, $\mu = .1$

$$g_{1C} = 13.9 \times 10^{-6}\,,\ g_{1C}^{-1} = 72\ K$$

$$g_{2C} = 33.67 \times 10^{-6}\,,\ g_{2C}^{-1} = 29.7\ K$$

$$h_{2C} = 11.39 \times 10^{-6}\,,\ h_{2C}^{-1} = 87.8\ K$$

From Eq. (27)

$$g_{1R} = 10.66 \times 10^{-6} \quad , \quad g_{1R}^{-1} = 93.9 \text{ K}$$

$$g_{2R} = 28.37 \times 10^{-6} \quad , \quad g_{2R}^{-1} = 35.25 \text{ K}$$

$$h_{1R} = .376 \times 10^{-6} \quad , \quad h_{1R}^{-1} = 2.66 \, \Omega$$

$$(k\, h_{1R})^{-1} = 1.73 \, \Omega$$

Finally, Eq. (14) gives for the virtual absorption in the reflector

$$\frac{\Sigma_{a2R}}{\Sigma_{s2R}} = .00172$$

and for the virtual fission Eq. (16) gives

$$\nu \frac{\Sigma_{f2R}}{\Sigma_{s2R}} = .00068$$

These quantities are negligible.

VI An Attempted Generalization

The correspondence scheme of Eq. (6.5) is not the most general that might be used, and it has seemed possible that by generalizing to

$$\Phi_1 = a_{11} V_1 - a_{12} V_2 \tag{31}$$

$$\Phi_2 = -a_{21} V_1 + a_{22} V_2 \tag{32}$$

the need for keeping μ small in order to minimize the contradictions of Eqs. (14) and (14.5) might be eliminated.

This hope has not been fulfilled. Eqs. (31) and (32) lead to a set of boundary conditions which cannot be satisfied save in the most exceptional circumstances.

Had this been possible, the limitations imposed by the need to have $\mu << 1$ would have been eliminated and it might have been possible to cascade several groups in the network. As it is, the practical voltage limits probably make it impractical to simulate more than 3 groups.

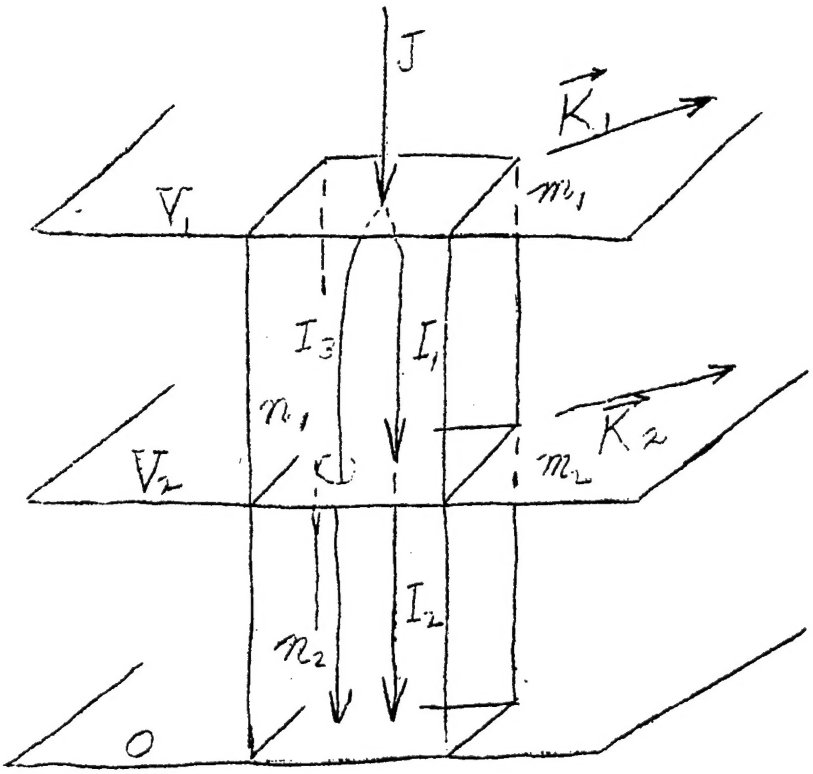

Figure 1

Electrical Analogue of 2-Group Analysis

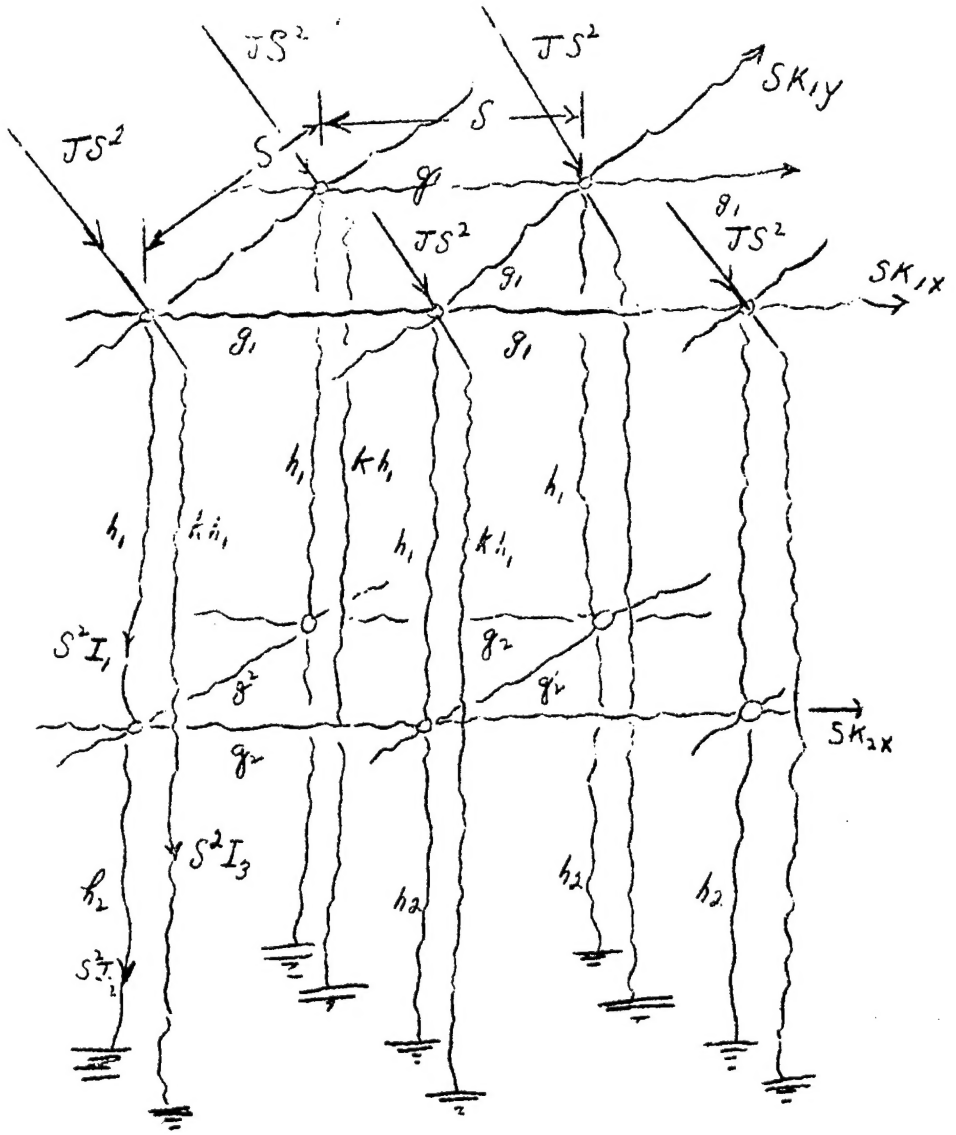

Figure 2.

Network Analogue of 2-Group Analysis

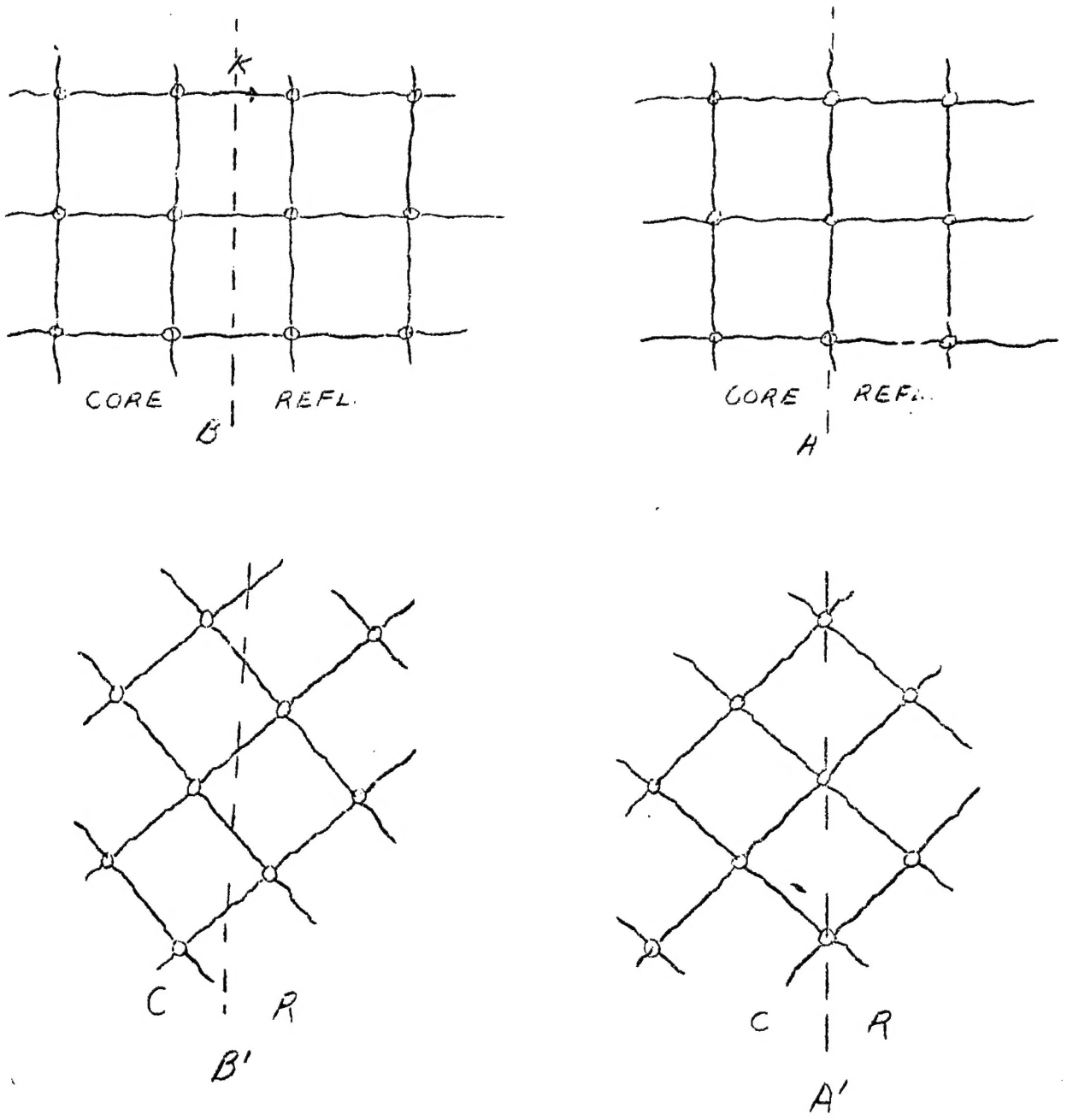

Figure 3.

Core-Reflector

Boundaries-Schematic

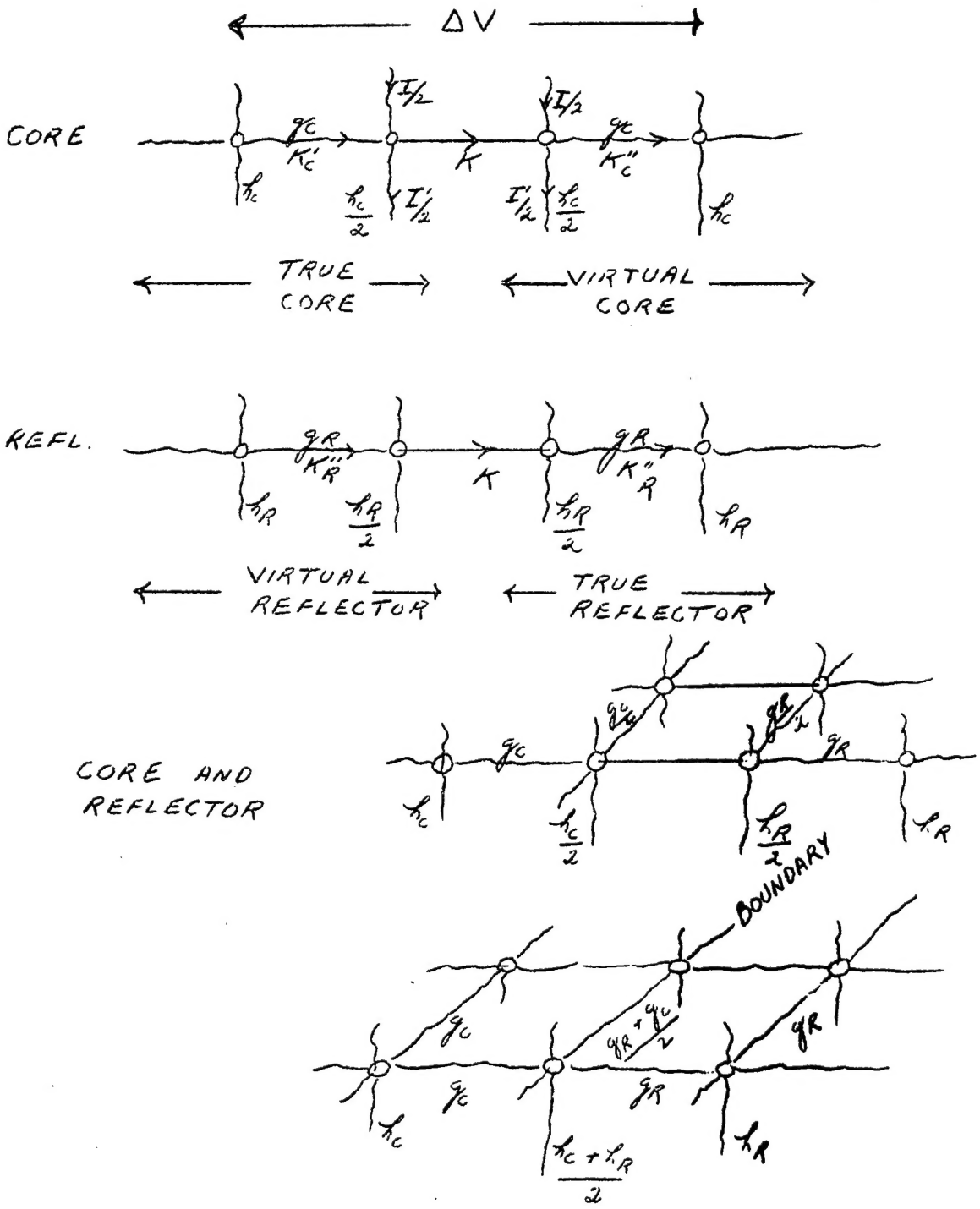

Figure 4.

Analysis of Core-Reflector
Boundary and Actual Boundary Arrangement

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